Bell's Theorem and Entangled Solitons

Yu.P. Rybakov[1] and T.F. Kamalov[2, 3, 4]

[1] *Theoretical Physics Department, Peoples' Friendship University of Russia, Moscow, Russian Federation*
[2] *Theoretical Physics Department, Moscow Institute of Physics and Technology, Moscow, Russian Federation*
[3] *Department of Physics, Moscow State University of Mechanical Engineering, Moscow, Russian Federation*
[4] *Theoretical Physics Department, Skolkovo Institute of Science and Technology, Moscow, Russian Federation*
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Entangled solitons construction being introduced in the nonlinear spinor field model, the Einstein—Podolsky—Rosen (EPR) spin correlation is calculated and shown to coincide with the quantum mechanical one for the 1/2–spin particles.

I.   INTRODUCTION

According to L. de Broglie [1] and A. Einstein [2], particles are considered as spatial regions with very high intensity of some fundamental field. Nowadays such field configurations are known as solitons. In this article we find new arguments in favour of the thought that the soliton concept can give a consistent description of extended quantum particles.

First of all we recall that the famous Bell's theorem [3] states that hypothetical hidden variables in quantum mechanics cannot be considered as local (point-like) ones. However, we intend to show that solitons prove to be considered as non-local (extended) hidden variables.

II.   ENTANGLED SOLITONS AND EPR CORRELATIONS

In the sequel we shall consider the special case of two–particles configurations corresponding to the singlet state of two 1/2–spin particles. In quantum mechanics these states are described by the spin wave function of the form

$$\psi_{12} = \frac{1}{\sqrt{2}} \left(|1 \uparrow\rangle \otimes |2 \downarrow\rangle - |1 \downarrow\rangle \otimes |2 \uparrow\rangle \right) \qquad (1)$$

and are known as **entangled states**. The arrows in (1) signify the projections of spin $\pm 1/2$ along some fixed direction. In the case of the electrons in the famous Stern—Gerlach experiment this direction is determined by that of an external magnetic field. If one chooses two different Stern—Gerlach devices, with the directions $\mathbf{a}$ and $\mathbf{b}$ of the magnetic fields, denoted by the unit vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, one can measure the correlation of spins of the two electrons by projecting the spin of the first electron on $\mathbf{a}$ and the second one on $\mathbf{b}$. Quantum mechanics gives for the spin correlation function the well-known expression

$$P(\mathbf{a}, \mathbf{b}) = \psi_{12}^{+}(\sigma\mathbf{a}) \otimes (\sigma\mathbf{b})\psi_{12}, \qquad (2)$$

where σ stands for the vector of Pauli matrices σ_i, $i = 1, 2, 3$. Putting (1) into (2), one easily gets

$$P(\mathbf{a}, \mathbf{b}) = -(\mathbf{ab}). \qquad (3)$$

The formula (3) characterizes the spin correlation in the Einstein—Podolsky—Rosen entangled singlet states and is known as the EPR–correlation. As was shown by J. Bell [3], the correlation (3) can be used as an efficient criterium for distinguishing the models with the local (point-like) hidden variables from those with the non-local ones. Namely, for the local-hidden-variables theories the EPR–correlation (3) is broken.

It would be interesting to check the solitonian model, shortly described in the beforehand points, by applying to it the EPR–correlation criterium. To this end let us first describe the 1/2–spin particles as solitons in the nonlinear spinor model of Heisenberg—Ivanenko type considered in the works [4, 5]. The soliton in question is described by the relativistic 4–spinor field φ of stationary type

$$\varphi = \begin{bmatrix} u \\ v \end{bmatrix} e^{-\imath\omega t}, \qquad (4)$$

satisfying the equation

$$\left(\imath\gamma^k \partial_k - \ell_0^{-1} + \lambda(\bar{\varphi}\varphi) \right) \varphi = 0, \qquad (5)$$

where u and v denote 2–spinors, k runs Minkowsky space indices 0, 1, 2, 3; ℓ_0 stands for some characteristic length (the size of the particle—soliton), λ is self-coupling constant, $\bar{\varphi} \equiv \varphi^{+}\gamma^0$, γ^k are the Dirac matrices. The stationary solution to the equation (5) can be obtained by separating variables in spherical coordinates r, ϑ, α via the substitution

$$u = \frac{1}{\sqrt{4\pi}} f(r) \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \frac{i}{\sqrt{4\pi}} g(r)\sigma_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (6)$$

where $\sigma_r = (\sigma\mathbf{r})/r$. Inserting (6) into (5), one finds

$$\frac{\omega}{c}u + \imath(\sigma\nabla)v - \ell_0^{-1}u + \frac{\lambda}{4\pi}\left(f^2 - g^2\right)u = 0,$$

$$\frac{\omega}{c}v + \imath(\sigma\nabla)u - \ell_0^{-1}v + \frac{\lambda}{4\pi}\left(f^2 - g^2\right)v = 0.$$

In view of (6) one gets

$$\imath(\sigma\nabla)v = -\frac{1}{\sqrt{4\pi}}\left(g' + \frac{2}{r}g\right)\begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\imath(\sigma\nabla)u = -\frac{1}{\sqrt{4\pi}}f'\sigma_r\begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Finally, one derives the following ordinary differential equations for the radial functions $f(r)$ and $g(r)$:

$$\left(g' + \frac{2}{r}g\right) = \left(\frac{\omega}{c} - \ell_0^{-1}\right)f + \frac{\lambda}{4\pi}\left(f^2 - g^2\right)f,$$

$$-f' = \left(\frac{\omega}{c} + \ell_0^{-1}\right)g + \frac{\lambda}{4\pi}\left(f^2 - g^2\right)g.$$

As was shown in the papers [4, 5], these equations admit regular solutions, if the frequency parameter ω belongs to the interval

$$0 < \omega < c/\ell_0. \tag{7}$$

The behavior of the functions $f(r)$ and $g(r)$ at $r \to 0$ is as follows:

$$g(r) = C_1 r, \quad f = C_2, \quad f' \to 0,$$

where C_1, C_2 denote some integration constants. The behavior of solutions far from the center of the soliton, i.e. at $r \to \infty$, is given by the relations:

$$f = \frac{A}{r}e^{-\nu r}, \quad g = -\frac{f'}{B},$$

where

$$\nu = \left(\ell_0^{-2} - \omega^2/c^2\right)^{1/2}, \quad B = \ell_0^{-1} + \omega/c.$$

If one chooses the free parameters ℓ_0 and λ of the model to satisfy the normalization condition

$$\int d^3x\, \varphi^+\varphi = \int_0^\infty dr\, r^2 \left(f^2 + g^2\right) = \hbar, \tag{8}$$

then the spin of the soliton reads

$$\mathbf{S} = \int d^3x\, \varphi^+ \mathbf{J}\varphi = \frac{\hbar}{2}\mathbf{e}_z, \tag{9}$$

where $\mathbf{e}_z$ denotes the unit vector along the Z–direction, $\mathbf{J}$ stands for the angular momentum operator

$$\mathbf{J} = -i[\mathbf{r}\triangledown] + \frac{1}{2}\,\sigma \otimes \sigma_0, \tag{10}$$

and σ_0 is the unit 2×2–matrix.

Now it is worth-while to show the positiveness of the energy E of the $1/2$–spin soliton. The energy E is given by the expression

$$E = c\int d^3x \left[-i\varphi^+(\alpha\triangledown)\varphi + \ell_0^{-1}\bar\varphi\varphi - \frac{\lambda}{2}(\bar\varphi\varphi)^2\right], \tag{11}$$

where $\alpha = \sigma \otimes \sigma_1$. The positiveness of the functional (11) emerges from the virial identities characteristic for the model in question. In fact, the equation for the stationary solution (4) can be derived from the variational principle based on the Lagrangian of the system

$$L = -E + \int d^3x\, \omega\varphi^+\varphi. \tag{12}$$

Performing the two-parameters scale transformation of the form $\varphi(x) \to \alpha\varphi(\beta x)$, one can derive from (12) and the variational principle $\delta L = 0$ the following two virial identities, which are valid for any regular stationary solution to the field equation (5):

$$\int d^3x\, [i\frac{2}{3}\varphi^+(\alpha\triangledown)\varphi + \frac{\omega}{c}\varphi^+\varphi - \ell_0^{-1}\bar\varphi\varphi + \frac{\lambda}{2}(\bar\varphi\varphi)^2] = 0, \tag{13}$$

$$\int d^3x\, [i\varphi^+(\alpha\triangledown)\varphi + \frac{\omega}{c}\varphi^+\varphi - \ell_0^{-1}\bar\varphi\varphi + \lambda(\bar\varphi\varphi)^2] = 0. \tag{14}$$

Using (13) and (14), one can express some sign-changing integrals through those of definite sign:

$$\int d^3x[-i\frac{1}{3}\varphi^+(\alpha\triangledown)\varphi] = \frac{\lambda}{2}\int d^3x\, (\bar\varphi\varphi)^2, \tag{15}$$

$$\int d^3x\, [\ell_0^{-1}\bar\varphi\varphi + \frac{\lambda}{2}(\bar\varphi\varphi)^2] = \frac{\omega}{c}\int d^3x\, \varphi^+\varphi. \tag{16}$$

Using the identities (15) and (16), one can represent the energy (11) of the soliton as follows:

$$E = c\int d^3x\, [\ell_0^{-1}\bar\varphi\varphi + \lambda(\bar\varphi\varphi)^2] = \omega\int d^3x\, \varphi^+\varphi = \hbar\omega, \tag{17}$$

where the normalization condition (8) was taken into account. Thus, one concludes, in the connection with (7) and (17), that the energy of the stationary spinor soliton (4) in the nonlinear model (5) turns out to be positive. Moreover, one can see that (17) is equivalent to the Planck—de Broglie wave—particle dualism relation. Now let us construct the two–particles singlet configuration on the base of the soliton solution (4). First of all, in analogy with (1), one constructs the entangled solitons configuration endowed with the zero spin:

$$\varphi_{12} = \frac{1}{\sqrt{2}}\left[\varphi_1^\uparrow \otimes \varphi_2^\downarrow - \varphi_1^\downarrow \otimes \varphi_2^\uparrow\right], \tag{18}$$

where $\varphi_1^\uparrow$ corresponds to (6) with $\mathbf{r} = \mathbf{r}_1$, and $\varphi_2^\downarrow$ emerges from the above solution by the substitution

$$\mathbf{r}_1 \to \mathbf{r}_2, \quad \begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$$

that corresponds to the opposite projection of spin on the Z–axis. In virtue of the orthogonality relation for the states with the opposite spin projections, one easily derives the following normalization condition for the entangled solitons configuration (18):

$$\int d^3x_1 \int d^3x_2\, \varphi_{12}^+\varphi_{12} = \hbar^2. \tag{19}$$

Now it is not difficult to find the expression for the stochastic wave function [6–8] for the singlet two–solitons state:

$$\Psi_N(t, \mathbf{r}_1, \mathbf{r}_2) = \left(\hbar^2 N\right)^{-1/2}\sum_{j=1}^N \varphi_{12}^{(j)}, \tag{20}$$

where $\varphi_{12}^{(j)}$ corresponds to the entangled soliton configuration in the j–th trial, with the number of trials $N \gg 1$.

Our final step is the calculation of the spin correlation (2) for the singlet two–soliton state. In the light of the fact that the operator σ in (2) corresponds to the twice angular momentum operator (10), one should calculate the following expression:

$$P'(\mathbf{a}, \mathbf{b}) = \mathbb{M} \int d^3x_1 \int d^3x_2 \, \Psi_N^+ 2 \left(\mathbf{J}_1 \mathbf{a} \right) \otimes 2 \left(\mathbf{J}_2 \mathbf{b} \right) \Psi_N, \tag{21}$$

where $\mathbb{M}$ stands for the averaging over the random phases of the solitons. Inserting (20) and (10) into (21), using the independence of trials $j \neq j'$ and taking into account the relations:

$$J_+\varphi^\uparrow = 0, \qquad J_3\varphi^\uparrow = \frac{1}{2}\varphi^\uparrow, \qquad J_-\varphi^\uparrow = \varphi^\downarrow,$$

$$J_-\varphi^\downarrow = 0, \qquad J_3\varphi^\downarrow = -\frac{1}{2}\varphi^\downarrow, \qquad J_+\varphi^\downarrow = \varphi^\uparrow,$$

where $J_\pm = J_1 \pm : iJ_2$, one easily finds that

$$P'(\mathbf{a}, \mathbf{b}) = -\hbar^{-2} \left(\mathbf{a}\mathbf{b} \right) \left(\int_0^\infty d\,r\,r^2 \left(f^2 + g^2 \right) \right)^2 = - \left(\mathbf{a}\mathbf{b} \right). \tag{22}$$

Comparing the correlations (22) and (3), one remarks their coincidence, that is the solitonian model satisfies the EPR–correlation criterium.

III. CONCLUSION

The coincidence of the quantum spin correlation with that in the solitonian scheme supports the hope that the latter one has many attractive features relevant to consistent theory of extended elementary particles [9–11].

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